

OSNOVE ELEKTROTEHNIKE I in II

Rešitve za Predlogo nalog za študijsko leto 2002/2003

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Pričujoče rešitve so v nastajanju in bodo dopolnjene ter popravljene. Da bi se to lahko odvijalo čim hitreje, vas prosimo, da vse napake in vprašanja sporočite na elektronski naslov (za 1., 2. in 4. poglavje *edi.bulic@fe.uni-lj.si*, za ostala poglavja *ales.berkopec@fe.uni-lj.si*).

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1 Elektrostatično polje

1.1.

$$\mathbf{e}_\rho = \mathbf{e}_x \frac{x}{\sqrt{x^2 + y^2}} + \mathbf{e}_y \frac{y}{\sqrt{x^2 + y^2}}, \quad \mathbf{e}_\varphi = \mathbf{e}_x \frac{-y}{\sqrt{x^2 + y^2}} + \mathbf{e}_y \frac{x}{\sqrt{x^2 + y^2}}$$

$$\mathbf{e}_x = \mathbf{e}_\rho \cos \varphi - \mathbf{e}_\varphi \sin \varphi, \quad \mathbf{e}_y = \mathbf{e}_\rho \sin \varphi + \mathbf{e}_\varphi \cos \varphi$$

1.2.

$$\begin{bmatrix} L_\rho \\ L_\varphi \end{bmatrix} = \begin{bmatrix} x/\sqrt{x^2 + y^2} & y/\sqrt{x^2 + y^2} \\ -y/\sqrt{x^2 + y^2} & x/\sqrt{x^2 + y^2} \end{bmatrix} \begin{bmatrix} L_x \\ L_y \end{bmatrix}$$

$$\begin{bmatrix} L_x \\ L_y \end{bmatrix} = \begin{bmatrix} \cos \varphi & -\sin \varphi \\ \sin \varphi & \cos \varphi \end{bmatrix} \begin{bmatrix} L_\rho \\ L_\varphi \end{bmatrix}$$

1.3.

1.4.

$$y = \frac{5k}{x} \implies V(x, y) = 10k, \quad k = 1, 2, 3, \dots$$

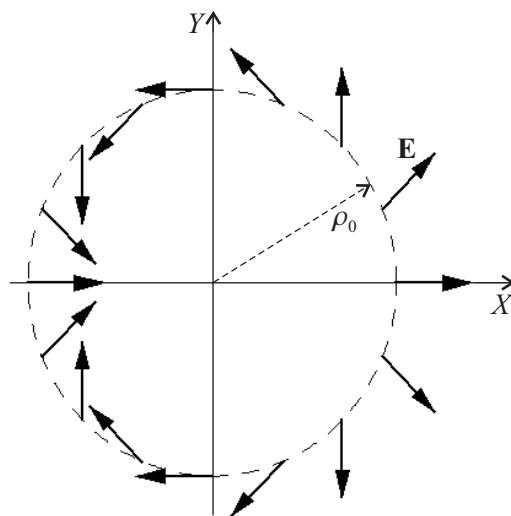
$$\cos \varphi = 10k\rho \implies U(\rho, \varphi) = 50k, \quad k = 1, 2, 3, \dots$$

$$V(\rho, \varphi) = \rho^2 \sin(2\varphi)$$

$$U(x, y) = \frac{5x}{x^2 + y^2}$$

1.5.

$$E(\rho_0, \varphi) = k/\rho_0^2 = \text{konst.}$$



1.6.

$$\bar{p} = \frac{1}{2} U_m I_m \cos(\alpha - \beta)$$

$$\bar{p} = 0 \text{ pri } \alpha - \beta = \pi/2 + k\pi, \quad k \in \mathbb{Z}$$

$$\bar{p} = \bar{p}_{\max} = \frac{1}{2} U_m I_m \text{ pri } \alpha - \beta = 2k\pi, \quad k \in \mathbb{Z}$$

1.7.

$$\int_{T_1}^{T_2} \mathbf{S} \cdot d\mathbf{l} = 30 \text{ in je neodvisen od poti med točkama } T_1 \text{ in } T_2.$$

1.8.

$$\int \mathbf{P} \cdot d\mathbf{a} = \pm 1$$

1.9.
$$\frac{\partial V}{\partial x} = \frac{\pi}{a} \cos \frac{\pi x}{a} \sinh \frac{\pi y}{a}, \quad \frac{\partial V}{\partial y} = \frac{\pi}{a} \sin \frac{\pi x}{a} \cosh \frac{\pi y}{a}$$

1.10. $\mathbf{F}_L = (-3, -10, 2) \text{ mN}, \quad F_L \doteq 10,6 \text{ mN}$

1.11. $Q(t_1) = 0,5 e^{-10 \text{ s}^{-1} t_1} \text{ A s}$

1.12. $n_e \doteq 6,24 \cdot 10^{12}$

1.13. $x = 10 \text{ mm}, \quad Q_x = -4 \mu\text{C}$

1.14. $Q_x = -Q/\sqrt{3}$

1.15.
$$Q_0 = -\frac{1 + 2\sqrt{2}}{4} Q$$

1.16.
$$F = \frac{\sqrt{3} Q^2}{2\sqrt{2} \pi \varepsilon_0 a^2}$$

$$F_{\min} = \frac{\sqrt{3} Q^2}{2\pi \varepsilon_0 a^2}$$

Ni formalne razlike, razlika je le v multiplikativni konstanti: $F_{\min} = \frac{Q^2}{6\sqrt{3}\pi\varepsilon_0\rho_0^2}$, kjer sta Q in ρ_0 naboj ter polmer prstana.

1.17.
$$a = \frac{1}{1 + \sqrt{2}} l, \quad b = \frac{\sqrt{2}}{1 + \sqrt{2}} l$$

1.18.
$$W_{k2} = \frac{Q_1 Q_2}{4\pi \varepsilon_0 d}$$

$$W_{k1} = \frac{Q_2 Q_1}{4\pi \varepsilon_0 d} = W_{k2}$$

1.19.
$$v_0 = \sqrt{\frac{e(-Q)}{2\pi \varepsilon_0 \rho_0 m}}$$

1.20.
$$W_e = \frac{qQ}{\pi \varepsilon_0} \ln 3$$

1.21.
$$\mathbf{f}_{e \text{ na } q_3} = -\mathbf{e}_x \frac{(q_1 + q_2)(q_1 + 2q_2)}{4\pi \varepsilon_0 a}$$

$$\mathbf{f}_{e \text{ na } q_3}(t) = \mathbf{e}_x \frac{q_0^2}{16\pi \varepsilon_0 a} (4 \cos(2\omega t) + 2 \cos(2\omega t - 2\pi/3) - 3), \quad \overline{\mathbf{f}_{e \text{ na } q_3}} = -\mathbf{e}_x \frac{3q_0^2}{16\pi \varepsilon_0 a}$$

1.22.

$$\mathbf{E}(z) = \begin{cases} -\mathbf{e}_z \rho d / (2\varepsilon_0) & , z < -d/2 \\ -\mathbf{e}_z \rho z / \varepsilon_0 & , -d/2 \leq z \leq d/2 \\ \mathbf{e}_z \rho d / (2\varepsilon_0) & , z > d/2 \end{cases}$$

Če bi plast nabojev 'stisnili' v ravnino, bi ta imela ploskovno gostoto elektrine enako produktu ρd . Ta naelektrena ravnina ustvari izven oblaka enako polje kot oblak sam.

Če imata elektrina oblaka in točkasti naboj množine Q enak predznak ($\rho Q > 0$):

$v_z \geq \frac{d}{2} \sqrt{\frac{Q\rho}{m\varepsilon_0}}$. V nasprotnem točkasti naboj vedno preleti oblak neglede, kolikšna je njegova hitrost tik pod oblakom v smeri osi Z .

1.23.

$$\mathbf{E}(x) = \begin{cases} \mathbf{e}_x \rho_0 (x_0 - |x|) / \varepsilon_0 & , |x| \leq x_0 \\ \mathbf{0} & , |x| > x_0 \end{cases}$$

$$V(x) - C = \begin{cases} \rho_0 x_0^2 / \varepsilon_0 & , x < -x_0 \\ \rho_0 (x_0^2 - 2x_0 x - x^2) / (2\varepsilon_0) & , -x_0 \leq x \leq 0 \\ \rho_0 (x_0 - x)^2 / (2\varepsilon_0) & , 0 < x \leq x_0 \\ 0 & , x > x_0 \end{cases}$$

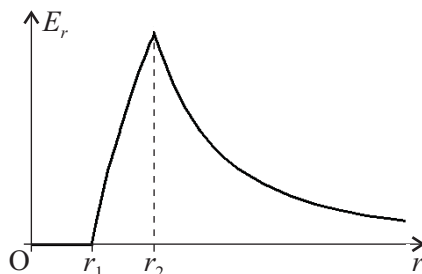
$$U = -\rho_0 x_0^2 / \varepsilon_0$$

1.24.

$$\mathbf{E}(\rho) = \begin{cases} \mathbf{e}_\rho \rho_e \rho / (2\varepsilon_0) & , \rho \leq \rho_0 \\ \mathbf{e}_\rho \rho_e \rho_0^2 / (2\varepsilon_0 \rho) & , \rho > \rho_0 \end{cases}$$

1.25.

$$\mathbf{E}(r) = \begin{cases} \mathbf{0} & , r < r_1 \\ \mathbf{e}_r \rho (r - r_1^3/r^2) / (3\varepsilon_0) & , r_1 \leq r \leq r_2 \\ \mathbf{e}_r \rho (r_2^3 - r_1^3) / (3\varepsilon_0 r^2) & , r > r_2 \end{cases}$$



$$d \ll r_2 \implies \mathbf{E}(r) = \begin{cases} \mathbf{0} & , r < r_2 \\ \mathbf{e}_r \sigma r_2^2 / (\varepsilon_0 r^2) & , r > r_2 \end{cases}$$

1.26.

$$\mathbf{E}(r) = \mathbf{e}_r \frac{\rho_0 r_0^3}{3\varepsilon_0 r^2} \left(1 - e^{-(r/r_0)^3} \right)$$

1.27.

$$\mathbf{E}(x) = \begin{cases} \mathbf{0} & , x < 0 \\ -\mathbf{e}_x \sigma / \varepsilon_0 & , 0 < x < a \\ \mathbf{e}_x 2\sigma / \varepsilon_0 & , a < x < a + b \\ \mathbf{0} & , x > a + b \end{cases}$$

1.28. $\mathbf{M}_e = Q\mathbf{d} \times \mathbf{E}_0$

- 1.29. Vektor sile na dolžinski meter ima smer radialno stran od težišča mnogokotnika, velikost pa $f_e = \frac{q^2(n-1)}{2\pi\epsilon_0 a} \sin \frac{\pi}{n}$, kjer je n število nabojev, q njihova vzdolžna elektrina in a dolžina stranic mnogokotnika.

1.30.

1.31. 1.25.

$$V(r) = \begin{cases} \rho(r_2^2 - r_1^2)/(2\epsilon_0) & , \quad r < r_1 \\ \rho(-r^2/2 + 3r_2^2/2 - r_1^3/r)/(3\epsilon_0) & , \quad r_1 \leq r \leq r_2 \\ \rho(r_2^3 - r_1^3)/(3\epsilon_0 r) & , \quad r > r_2 \end{cases}$$

1.27. Potencial je le do konstante (C) natančno določen:

$$V(x) = \begin{cases} \sigma(2b-a)/\epsilon_0 + C & , \quad x \leq 0 \\ \sigma(2b-a+x)/\epsilon_0 + C & , \quad 0 < x \leq a \\ 2\sigma(a+b-x)/\epsilon_0 + C & , \quad a < x \leq a+b \\ C & , \quad x > a+b \end{cases}$$

1.32. $-\Delta W_{\text{ep}} = Q(dE_0 - \mathbf{d} \cdot \mathbf{E}_0)$

$$W_{\text{ep}} = Q(dE_0 - \mathbf{d} \cdot \mathbf{E}_0)$$

1.33.

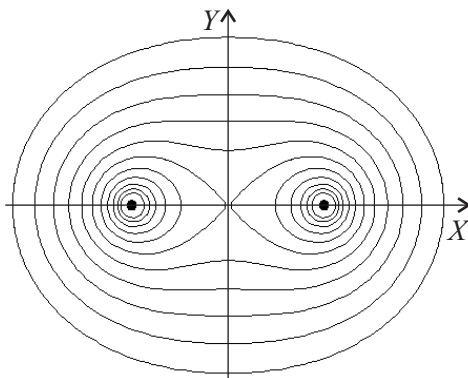
$$V(x, y) = \frac{q}{2\pi\epsilon_0} \ln \frac{\sqrt{(x-s)^2 + y^2}}{\sqrt{(x+s)^2 + y^2}}$$

$$\mathbf{E}(x, y) = \frac{q}{2\pi\epsilon_0} \left[\mathbf{e}_x \left(\frac{x+s}{(x+s)^2 + y^2} - \frac{x-s}{(x-s)^2 + y^2} \right) + \mathbf{e}_y \left(\frac{y}{(x+s)^2 + y^2} - \frac{y}{(x-s)^2 + y^2} \right) \right]$$

$$s \ll \sqrt{x^2 + y^2} \implies V(x, y) = -\frac{qs}{\pi\epsilon_0} \frac{x}{x^2 + y^2}$$

$$\mathbf{E}(x, y) = -\frac{qs}{\pi\epsilon_0} \frac{1}{(x^2 + y^2)^2} [\mathbf{e}_x (x^2 - y^2) + \mathbf{e}_y 2xy]$$

1.34.



- 1.35. Če je $|Q_1| = |Q_2|$, je ekvipotencialka $V = 0$ V simetralna ravnina med elektrinama. Če je $|Q_1| > |Q_2|$, je ekvipotencialka $V = 0$ V sfera polmera $\frac{l}{|Q_1|/|Q_2| - |Q_2|/|Q_1|}$, katere središče se nahaja na premici, ki gre skozi obe elektrini, in je od elektrine Q_2 oddaljeno za $\frac{l}{|Q_1|^2/|Q_2|^2 - 1}$ proti desni.

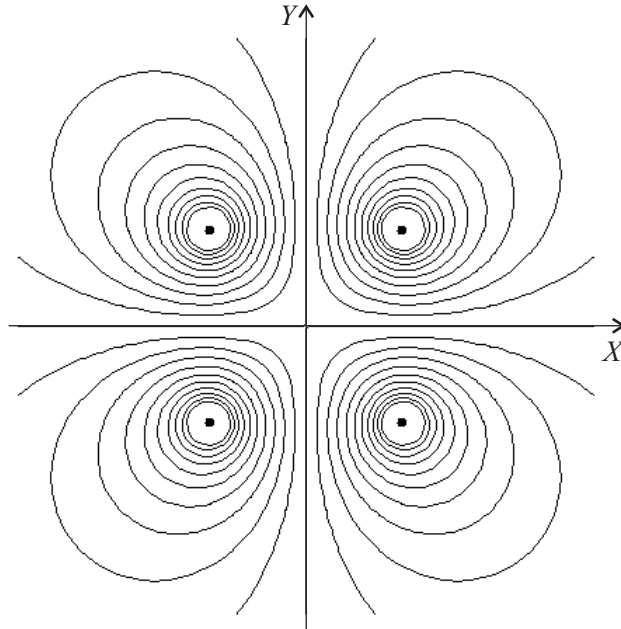
1.36. Ekvipotencialne ploskve so valjne lupine soosne nabojev s polmeri

$$\rho_i = b(a/b)^{i/10}, \quad i = 1, 2, \dots, 9.$$

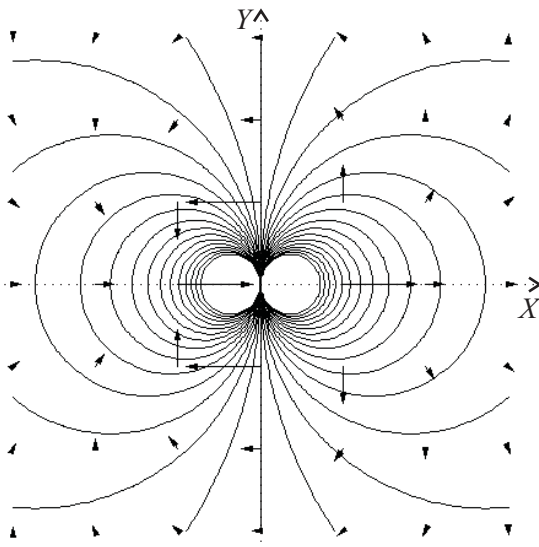
1.37. $Q_2 = -2Q_1$

1.38.

$$V(T) = \frac{q}{2\pi\epsilon_0} \ln \frac{\rho_2\rho_4}{\rho_1\rho_3}$$



- 1.39. Ekvipotencialke so valjne lupine (v prerezni ravnini krožnice), ki se dotikajo osi Y , njihove osi pa so vzporedne premima elektrinama in sekajo os X . Oddaljenosti teh osi do premega dipola so enake polmerom ekvipotencialnih valjnih lupin.



$E^2 = \left(\frac{p}{2\pi\epsilon_0\rho^2} \right)^2$; konstanten je na valjnih lupinah (v prerezni ravnini krožnicah) soosnih prememu dipolu.

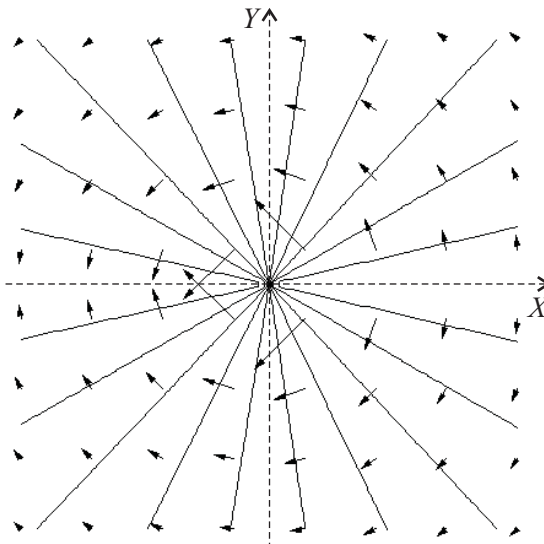
$$\int_{\rho=0}^{\rho=a} E^2 dv = \frac{p^2 l}{4\pi\epsilon_0^2 a^2}$$

- 1.40. a) Vz dolžna gostota leve elektrine je $q_l \doteq 21,1 \text{ nC/m}$, njena ekscentričnost pa $e_l \doteq 1,34 \text{ cm}$. Podobno sta za desno elektrino $q_d = -q_l$ in $e_d = e_l$.
- b) $q_l = -q_d \doteq -3,16 \text{ } \mu\text{C/m}$. Oddaljenosti elektrin do leve ekvipotencialke sta $s \doteq 28,3 \text{ cm}$.
- c) $q_l = -q_d \doteq -243 \text{ nC/m}$, $e_l \doteq 2,10 \text{ cm}$ in $e_d \doteq 0,60 \text{ cm}$.
- d) $q_l = -q_d \doteq 5,77 \text{ nC/m}$. Elektrini se nahajata približno $0,15 \text{ mm}$ oz. $6,85 \text{ mm}$ desno od središča notranje ekvipotencialke.

1.41.

$$\mathbf{E} = (-a, -b, -c), \quad E = \sqrt{a^2 + b^2 + c^2}, \quad \delta l = \frac{U}{\sqrt{a^2 + b^2 + c^2}}$$

1.42.



1.43.

1.44.

$$\mathbf{M}_1 = \frac{\mathbf{p}_1 \times \mathbf{p}_2}{2\pi\epsilon_0 a^3}, \quad \mathbf{M}_2 = \frac{\mathbf{p}_1 \times \mathbf{p}_2}{4\pi\epsilon_0 a^3}$$

Načelo vzajemnosti za navora ne velja, saj je $\mathbf{M}_1 \neq -\mathbf{M}_2$.

1.45. $Q \geq 12\pi\epsilon_0 r_0^2 E_0$

1.46.

$$E_\rho(\rho) \doteq \begin{cases} 910 \text{ V}/\rho & , \rho_n < \rho < \rho_z \\ 0 & , \rho < \rho_n \text{ ali } \rho > \rho_z \end{cases}$$

$$V(\rho) \doteq \begin{cases} 1 \text{ kV} & , \rho \leq \rho_n \\ (910 \text{ V}) \ln(3 \text{ mm}/\rho) & , \rho_n < \rho < \rho_z \\ 0 & , \rho \geq \rho_z \end{cases}$$

Gostoti elektrin na stenah žile in plašča sta $\sigma_n \doteq 8,06 \mu\text{C}/\text{m}^2$ in $\sigma_z \doteq -2,69 \mu\text{C}/\text{m}^2$, množini elektrin pa sta $Q_n \doteq 5,06 \mu\text{C}$ in $Q_z \doteq -5,06 \mu\text{C}$.

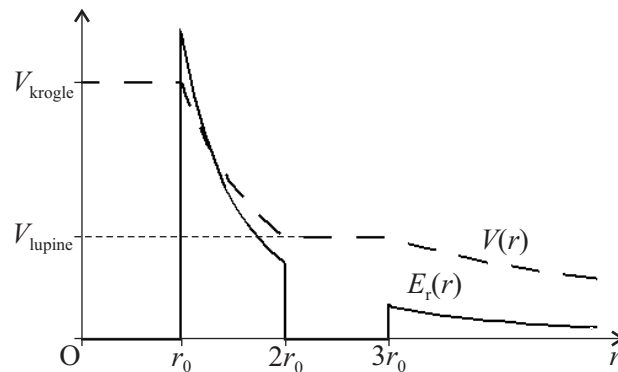
1.47. $Q \doteq 32,3 \text{ nC}$

1.48.

$$V_{\text{krogla}} = \frac{5Q}{24\pi\epsilon_0 r_0}, \quad V_{\text{lupine}} = \frac{Q}{12\pi\epsilon_0 r_0}$$

$$E_r(r) = \begin{cases} 0 & , r < r_0 \\ Q/(4\pi\epsilon_0 r^2) & , r_0 < r < 2r_0 \\ 0 & , 2r_0 < r < 3r_0 \\ Q/(4\pi\epsilon_0 r^2) & , r > 3r_0 \end{cases}$$

$$V(r) = \begin{cases} 5Q/(24\pi\epsilon_0 r_0) & , r \leq r_0 \\ Q/(4\pi\epsilon_0 r) - Q/(24\pi\epsilon_0 r_0) & , r_0 < r \leq 2r_0 \\ Q/(12\pi\epsilon_0 r_0) & , 2r_0 < r \leq 3r_0 \\ Q/(4\pi\epsilon_0 r) & , r > 3r_0 \end{cases}$$



1.49. Če izberemo koordinatno izhodišče na levi plošči in če je ozemljena negativna sponka vira, potem velja:

$$\mathbf{E}(x) \doteq \begin{cases} \mathbf{e}_x 33,3 \text{ kV/m} & , 0 < x < 3 \text{ mm} \\ \mathbf{0} & , \text{ drugod} \end{cases},$$

$$V(x) \doteq 100 \text{ V} - (33,3 \text{ kV/m})x, \quad 0 \leq x \leq 3 \text{ mm}.$$

Ko je vir priključen med srednjo in obe krajni plošči, ki sta galvanjsko povezani in ozemljeni, velja:

$$\mathbf{E}(x) = \begin{cases} -\mathbf{e}_x 100 \text{ kV/m} & , 0 < x < 1 \text{ mm} \\ \mathbf{e}_x 50 \text{ kV/m} & , 1 \text{ mm} < x < 3 \text{ mm} \\ \mathbf{0} & , \text{ drugod} \end{cases},$$

$$V(x) = \begin{cases} (100 \text{ kV/m})x & , 0 \leq x \leq 1 \text{ mm} \\ (50 \text{ kV/m})(3 \text{ mm} - x) & , 1 \text{ mm} < x \leq 3 \text{ mm} \\ 0 & , \text{ drugod} \end{cases}.$$

1.50.

1.51. $q = 2\pi a \sigma_0$

1.52.

$$A_z = \frac{Q^2}{32\pi\epsilon_0 a}$$

1.53. $\mathbf{M}_e = \mathbf{e}_z 10^{-3} \text{ N m}$

1.54. $q_1/q \doteq 35\%$, $q_2/q \doteq 30\%$

1.55. Na krajnih žicah je (približno) po 30,6% celotnega naboja, na srednjih pa (približno) po 19,4%.

1.56. $\rho_0 = 2^{1/3} \sqrt{4^{1/3} + 1} h \doteq 2,03h$

$$Q_0 \doteq 0,260Q \simeq Q/4$$

1.57. $\sigma(T) \doteq -2,55 \text{ nC/m}^2$

1.58.

$$\sigma(d) = -\frac{2\varepsilon_0 h U}{\ln(2h/\rho_0)} \frac{1}{d^2}, \text{ kjer je } d \text{ oddaljenost (točk na površini zemlje) od osi vodnika.}$$

1.59.

$$V_1 \doteq 3,65 \frac{q}{2\pi\varepsilon_0}, \quad V_2 \doteq -6,12 \frac{q}{2\pi\varepsilon_0}$$

1.60. $U \doteq 1,40 \text{ kV}$

1.61. $\sigma_{\max}/\sigma_{\min} = 27$

Če izberemo koordinatni sistem tako, da je izhodišče v središču lupine ter da os Z kaže v desno, potem velja:

$$\sigma(z) = \frac{Q}{4\pi r_0^2} \frac{6}{(5 - 4z/r_0)^{3/2}},$$

$$\mathbf{F} = \mathbf{e}_z \frac{2Q^2}{9\pi\varepsilon_0 r_0^2}.$$

Če je lupina nevtralna, je njen potencial $V = \frac{Q}{4\pi\varepsilon_0 r_0}$.

Točkast naboj ne "opazi" razlike, če je lupina ozemljena ali če je nevtralna, zato ker sta notranjost in zunanost lupine zaključena avtonomna električna sistema. Sila na ta naboj je namreč v obeh primerih enaka.

1.62.

$$V = \frac{Qx(d-x)}{\varepsilon_0 S d}$$

$$Q_x = -Q \frac{d-x}{d}$$

$$Q_{d-x} = -Q \frac{x}{d}$$

1.63. $V \doteq 899 \text{ V}$

1.64.

$$\sigma(y) \doteq \frac{-645 \text{ pC/m}^2}{1,6 \cdot 10^{-3} + y^2/\text{m}^2}, \quad \frac{\sigma_{\max}}{\sigma_{\min}} = 4$$

$$Q \doteq \mp 506 \text{ nC}$$

$$Q_{\text{prednji}} \doteq 70,5 \% Q_{\text{celotni}}, \quad Q_{\text{hrbtni}} \doteq 29,5 \% Q_{\text{celotni}}$$

1.65. Koordinatni sistem izberimo (npr.) tako, da osi X in Y sovpadata vsaka z eno steno vogala ter da obe kažeta proti zunanosti vogala, os Z pa naj je vzporedna z vogalom.

$$\mathbf{f}_e = -(\mathbf{e}_x + \mathbf{e}_y) \frac{q^2}{8\pi\varepsilon_0 a} = -\left(\frac{\mathbf{e}_x}{\sqrt{2}} + \frac{\mathbf{e}_y}{\sqrt{2}}\right) \frac{q^2}{4\sqrt{2}\pi\varepsilon_0 a}$$

$$\sigma(x) = -\frac{4qa^2}{\pi} \frac{x}{x^4 + 4a^4}, \quad \sigma(y) = -\frac{4qa^2}{\pi} \frac{y}{y^4 + 4a^4}$$

1.66. $q \doteq 387 \text{ nC/m}$

1.67. $Q \doteq 604 \text{ nC}$

Lahko merimo jakost homogenega polja tako, da napravo potem, ko jo v tem polju naelektrimo, prestavimo v prostor ničelnega polja (npr. Faradayeva kletka) ter pomerimo napetost $U_{E=0}$ med vodnikoma: $E_0 \doteq U_{E=0}/d$.

Bolj primerni bi bili prevodni kroglici namesto (dolgih) vodnikov, ker je potem območje v katerem merimo polje manjše pa v njem lažje dosežemo homogenost polja.

Dobili bi harmonični tokovni vir.

1.68.

Sila je odbojna velikosti $F = \frac{11Q^2}{288\pi\epsilon_0 r_0^2}$, kjer je Q množina točkastega naboja (tudi naboja krogle) ter r_0 polmer krogle.

$$A = \frac{11Q^2}{96\pi\epsilon_0 r_0}$$

1.69. $V = 550 \text{ MV}$, $\rho \doteq 88,5 \text{ nC/m}^3$

1.70.

$$E_r(r_1) = U \frac{r_2/r_1}{r_2 - r_1}, \quad r_{10} = 1 \text{ cm}$$

1.71. Os X koordinatnega sistema izberimo (npr.) tako, da je ta pravokotna na ravnini ter da ima izhodišče na eni ravnini in kaže proti drugi.

$$\mathbf{E}(x) = \mathbf{e}_x \frac{\rho}{2\epsilon_0} (2x - d), \quad V(x) = \frac{\rho}{2\epsilon_0} x(d - x)$$

$$\sigma = -\frac{\rho d}{2}$$

1.72.

$$U_0 = \frac{U \ln 2}{2 \ln(a/b)}$$

Medosno razdaljo prvega para žic (dvovoda), med katerima je napetost U , označimo z a_1 . Medosno razdaljo med pozitivnima žicama dvovodov označimo z d_1 , med negativnima pa z d_2 . Medosni razdalji med pozitivno žico enega dvovoda in negativno drugega označimo z D_1 oz. D_2 : $U_0 = \frac{U \ln(D_1 D_2 / (d_1 d_2))}{2 \ln(a_1/b)}$.

$D_1/d_1 = d_2/D_2 \implies U_1 = 0$, ker v tem primeru prvi dvovod povzroča enak potencial na obeh žicah drugega.

1.73. Izberimo (npr.) valjni koordinatni sistem, ki mu os Z postavimo pravokotno na podlago, tako da kaže v zunanost ter da je izhodišče na površini podlage.

$$\sigma(\rho) = \frac{\mathbf{p} \mathbf{e}_z}{2\pi} \frac{2h^2 - \rho^2}{(h^2 + \rho^2)^{5/2}}, \text{ kjer je } \mathbf{p} \text{ moment dipola ter } h \text{ njegova višina nad podlago.}$$

Matematične težave se pojavijo zato, ker ni več rotacijske simetrije in je $\sigma = \sigma(\rho, \varphi)$.

1.74.

- 1.75. Izberimo (npr.) valjni koordinatni sistem, kjer os Z sovpada z osjo curka in cilindra. Z ρ_c označimo volumsko gostoto elektrine v curku, z a polmer curka in z b notranji polmer cilindra.

$$\mathbf{E}(\rho) = \begin{cases} \mathbf{e}_\rho \rho_c \rho / (2\varepsilon_0) & , \quad \rho \leq a \\ \mathbf{e}_\rho \rho_c a^2 / (2\varepsilon_0 \rho) & , \quad a < \rho < b \end{cases}$$

$$V(\rho) = \begin{cases} \rho_c [a^2(1 + \ln(b/a)) - \rho^2] / (4\varepsilon_0) & , \quad \rho \leq a \\ \rho_c a^2 \ln(b/\rho) / (2\varepsilon_0) & , \quad a < \rho \leq b \end{cases}$$

- 1.76.

$$\sigma(r, \vartheta) = \begin{cases} -3\varepsilon_0 E_0 \cos \vartheta & , \quad r = r_0 \text{ in } \vartheta \in [0, \pi/2] \\ -\varepsilon_0 E_0 [1 - (r_0/r)^3] & , \quad r > r_0 \text{ in } \vartheta = \pi/2 \end{cases}$$

$$E_{\max} = E(r \rightarrow r_0 + 0, \vartheta = 0) = 3E_0, \quad E_{\min} = E(r = r_0, \vartheta \rightarrow \pi/2 - 0) = 0$$

- 1.77. $V = 2^{2/3} \cdot 100 \text{ V} \doteq 159 \text{ V}$

- 1.78.

- 1.79.

$$V(x, y) = \frac{V_0/2}{\ln \frac{2b(b^2 - a^2)}{c(b^2 + a^2)}} \ln \frac{[x^2 + (y - a^2/b)^2][x^2 + (y + b)^2]}{[x^2 + (y - b)^2][x^2 + (y + a^2/b)^2]}$$

$$\mathbf{E}(x, y) = \frac{V_0}{\ln \frac{2b(b^2 - a^2)}{c(b^2 + a^2)}} \left[\frac{1}{x^2 + (y - b)^2} (\mathbf{e}_x x + \mathbf{e}_y (y - b)) + \frac{-1}{x^2 + (y - a^2/b)^2} (\mathbf{e}_x x + \mathbf{e}_y (y - a^2/b)) + \frac{1}{x^2 + (y + a^2/b)^2} (\mathbf{e}_x x + \mathbf{e}_y (y + a^2/b)) + \frac{-1}{x^2 + (y + b)^2} (\mathbf{e}_x x + \mathbf{e}_y (y + b)) \right]$$

- 1.80.

- 1.81.

$$V = \frac{Q}{(2\sqrt{2} - 1)2\sqrt{2}\pi\varepsilon_0 r_0}$$

- 1.82. $E_{\text{zrak}} = 75 \text{ kV/m}$

- 1.83. Če je $U = 11 \text{ kV}$, potem velja:

$$\mathbf{E}(x) = \begin{cases} \mathbf{e}_x (1 \text{ kV/cm}) & , \quad 0 < x < 2 \text{ cm} \\ \mathbf{e}_x (3 \text{ kV/cm}) & , \quad 2 \text{ cm} < x < 5 \text{ cm} \end{cases}$$

$$V(x) = \begin{cases} (11 \text{ cm} - x) \cdot 1 \text{ kV/cm} & , \quad 0 \leq x \leq 2 \text{ cm} \\ (5 \text{ cm} - x) \cdot 3 \text{ kV/cm} & , \quad 2 \text{ cm} < x \leq 5 \text{ cm} \end{cases}$$

$$\sigma_{\text{prosti}}(x = 0) \doteq 2,66 \text{ } \mu\text{C/m}^2, \quad \sigma_{\text{prosti}}(x = 5 \text{ cm}) \doteq -2,66 \text{ } \mu\text{C/m}^2,$$

$$\sigma_{\text{vezani}}(x = 0) \doteq -1,77 \text{ } \mu\text{C/m}^2, \quad \sigma_{\text{vezani}}(x = 2 \text{ cm}) \doteq 1,77 \text{ } \mu\text{C/m}^2.$$

$$U_{\text{preb.}} \doteq 106 \text{ kV}$$

- 1.84. $Q = 0 \text{ A s}$

1.85.

$$\phi_e/U = Q/U = C = \frac{S}{\sum_{i=1}^n \frac{d_i}{\varepsilon_i}}$$

$$\phi_e/U = Q/U = C = \frac{\varepsilon_0 S}{d} \frac{e}{e-1}$$

$$x_k = d[1 - \ln(k(e-1) + 1)]$$

 1.86. $V(T) = 360 \text{ V}$

 1.87. Izberimo (npr.) valjni koordinatni sistem, kjer os Z sovpada z osjo palice in cevi. Polja med palico in cevjo ($a < \rho < b$) so:

$$\mathbf{E}(\rho) = \mathbf{e}_\rho \frac{V_0}{\ln b/a} \frac{1}{\rho}, \quad V(\rho) = V_0 \frac{\ln b/\rho}{\ln b/a},$$

$$\mathbf{D}(\rho) = \begin{cases} \mathbf{e}_\rho (\varepsilon_0 V_0) / (\rho \ln b/a) & , \text{ v zraku} \\ \mathbf{e}_\rho (4\varepsilon_0 V_0) / (\rho \ln b/a) & , \text{ v olju} \end{cases},$$

$$\mathbf{P}(\rho) = \begin{cases} \mathbf{0} & , \text{ v zraku} \\ \mathbf{e}_\rho (3\varepsilon_0 V_0) / (\rho \ln b/a) & , \text{ v olju} \end{cases}.$$

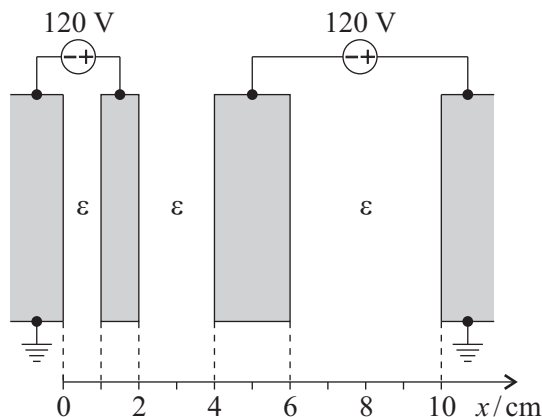
 1.88. Izberimo (npr.) krogelni koordinatni sistem, ki ima izhodišče v središču kroglice. Polja izven kroglice ($r > r_0$) so:

$$\mathbf{D}(r) = \mathbf{e}_r \frac{Q}{4\pi r^2}, \quad r > r_0,$$

$$\mathbf{E}(r) = \begin{cases} \mathbf{e}_r Q / (8\pi \varepsilon_0 r^2) & , \quad r_0 < r < 2r_0 \\ \mathbf{e}_r Q / (4\pi \varepsilon_0 r^2) & , \quad r > 2r_0 \end{cases},$$

$$V(r) = \begin{cases} Q(1/r + 1/(2r_0)) / (8\pi \varepsilon_0) & , \quad r_0 \leq r \leq 2r_0 \\ Q / (4\pi \varepsilon_0 r) & , \quad r > 2r_0 \end{cases}.$$

1.89.



1.90.

$$\lim_{\rho \rightarrow \rho_0 - 0} E_t = \lim_{\rho \rightarrow \rho_0 + 0} E_t = -\frac{2}{\varepsilon_r + 1} E_0 \sin \varphi$$

$$\lim_{\rho \rightarrow \rho_0 - 0} D_n = \lim_{\rho \rightarrow \rho_0 + 0} D_n = \frac{2\varepsilon_r \varepsilon_0}{\varepsilon_r + 1} E_0 \cos \varphi$$

- 1.91. Če izberemo koordinatno izhodišče na levi plošči in če je ozemljena negativna sponka vira, potem velja:

$$\mathbf{E}(x) \doteq \begin{cases} \mathbf{e}_x 45,5 \text{ kV/m} & , \quad 0 < x < 1 \text{ mm} \\ \mathbf{e}_x 27,3 \text{ kV/m} & , \quad 1 < x < 3 \text{ mm} \\ \mathbf{0} & , \quad \text{drugod} \end{cases}$$

$$\mathbf{P}(x) \doteq \begin{cases} \mathbf{e}_x 805 \text{ nC/m}^2 & , \quad 0 < x < 1 \text{ mm} \\ \mathbf{e}_x 966 \text{ nC/m}^2 & , \quad 1 < x < 3 \text{ mm} \\ \mathbf{0} & , \quad \text{drugod} \end{cases}$$

$$\mathbf{D}(x) \doteq \begin{cases} \mathbf{e}_x 1,21 \text{ } \mu\text{C/m}^2 & , \quad 0 < x < 3 \text{ mm} \\ \mathbf{0} & , \quad \text{drugod} \end{cases}$$

$$V(x) \doteq \begin{cases} 100 \text{ V} - (45,5 \text{ kV/m})x & , \quad 0 \leq x \leq 1 \text{ mm} \\ 81,8 \text{ V} - (27,3 \text{ kV/m})x & , \quad 1 \leq x \leq 3 \text{ mm} \end{cases}$$

$$\sigma \doteq \pm 1,21 \text{ } \mu\text{C/m}^2$$

1.92.

1.93. $V_{\text{zgornje}} = 800 \text{ V}$, $V_{\text{spodnje}} = -200 \text{ V}$

- 1.94. Izberimo (npr.) krogelni koordinatni sistem z izhodiščem v središču mehurčka, os Z pa naj je usmerjena v smeri polja v mehurčku.

$$\mathbf{E}(r \rightarrow 1 \text{ mm} + 0, \vartheta) = \mathbf{e}_r (2,5 \text{ MV/m}) \cos \vartheta - \mathbf{e}_\vartheta (10 \text{ MV/m}) \sin \vartheta$$

1.95. $U_{12} = 15 \text{ V}$

1.96. $U_{12} = 7E_0 d$

- 1.97. Vstopati mora pod kotom $\arcsin(1/\sqrt{1 + \varepsilon_r})$ glede na normalo na površino lističa.

1.98. $c \doteq 10,4 \text{ pF/m}$

1.99. $C_{12} \doteq 6,05 \text{ nF}$, $C_{10} \doteq 7,85 \text{ nF}$, $C_{20} \doteq 8,05 \text{ nF}$

$$q_1 \doteq 889 \text{ nC/m}, \quad q_2 \doteq -309 \text{ nC/m}, \quad q_0 \doteq -580 \text{ nC/m}$$

$$V_1 \doteq 51 \text{ kV}, \quad V_2 \doteq -49 \text{ kV}$$

1.100. $Q \doteq \pm 24,4 \text{ mC}$

1.101.

$$C(x) = \frac{\varepsilon_0 S d}{x(d-x)}$$

1.102. $\Delta C/C = 60 \%$

1.103. $V(x = 2,5 \text{ mm}) = 6 \text{ kV}$

$$\sigma(x = 2,5 \text{ mm}) \doteq 35,4 \text{ } \mu\text{C/m}^2$$

$$C \doteq 59,0 \text{ pF}$$

1.104. $\Delta C/C \doteq 12,5 \%$

1.105.

$$C = \frac{\pi \varepsilon_0 l}{\ln(\sqrt{8}a/(3r))}$$

1.106. $C \doteq 98,2 \text{ pF}$

$$\Delta C/C \doteq 5,8 \%$$

1.107.

1.108. $c \doteq 5,49 \text{ pF/m}$

$$Q_1/Q \doteq 61,4 \%, \quad Q_2/Q \doteq 38,6 \%$$

1.109. $c \doteq 11,3 \text{ pF/m}$ 1.110. $q \doteq -1,9 \text{ } \mu\text{C/m}, \quad E \doteq 28,1 \text{ kV/m}$ 1.111. $C \doteq 79,1 \text{ pF}$

1.112.

1.113. $C_{\text{nad.}} \doteq 3,02 \text{ nF}$ 1.114. $C \doteq 1 \text{ nF}$

1.115.

1.116. $\Delta Q/Q = (\varepsilon_r - 1) \cdot 100 \%$, kjer je ε_r relativna dielektričnost lističa.1.117. $5/4 = 1,25$

$$\alpha \doteq 35,3^\circ$$

1.118. $Q = 2\varepsilon_0 S V_0 / x_0, \quad W_e = 4\varepsilon_0 S V_0^2 / (3x_0)$ 1.119. $W_e = \frac{1}{2} C V_0 (V_0 - V_1)$ 1.120. $W_e \doteq 403 \text{ J}$

1.121.

$$W_e = \frac{2\pi\rho^2 r_0^5}{45\varepsilon_0}$$

1.122.

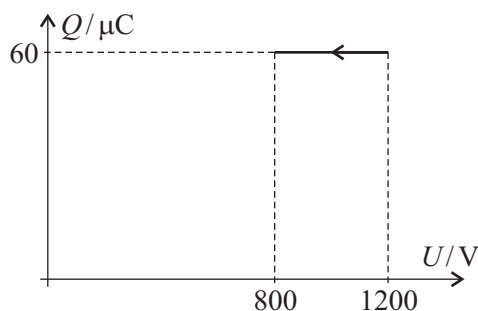
$$F_e = \frac{Q^2}{2\pi\varepsilon_0 l \sqrt{d^2 - 4\rho_0^2}}$$

1.123. $l = 3/4$. Minimum.

1.124.

1.125.

1.126.



$$A_e = 12 \text{ mJ}$$

1.127. $h \doteq 2,05 \text{ mm}$

1.128.

1.129.

1.130.

1.131. Na levem zgornjem kondenzatorju sta $Q_1 = 4Q_0/11$ in $U_1 = 4Q_0/(11C)$, na desnem zgornjem sta $Q_2 = 4Q_0/11$ in $U_2 = 2Q_0/(11C)$ ter na spodnjem $Q_3 = 18Q_0/11$ in $U_3 = 6Q_0/(11C)$.

$$\Delta W_e/W_e \doteq -40,5 \%$$

1.132. $U_1 = -700 \text{ V}$, $U_2 = 200 \text{ V}$, $U_3 = 500 \text{ V}$

1.133. Referenčne smeri napetosti na kondenzatorjih izberimo (npr.) navzdol.

$$U_{C_i} = -U_i + \frac{\sum_{j=1}^n C_j U_j}{\sum_{j=1}^n C_j}, \quad i = 1, 2, \dots, n$$

$$Q_i = C_i U_{C_i} = C_i \left(-U_i + \frac{\sum_{j=1}^n C_j U_j}{\sum_{j=1}^n C_j} \right), \quad i = 1, 2, \dots, n$$

$$W_{e,i} = C_i U_{C_i}^2 = C_i \left(U_i - \frac{\sum_{j=1}^n C_j U_j}{\sum_{j=1}^n C_j} \right)^2, \quad i = 1, 2, \dots, n$$

1.134. $\alpha = 40^\circ$

1.135. $V_A = -7 \text{ V}$

Kondenzatorje oštevilčimo (npr.) od zgoraj navzdol. Referenčne smeri napetosti na njih izberimo (npr.) od leve proti desni.

$$U_1 = -4 \text{ V}, \quad U_2 = -1 \text{ V}, \quad U_3 = 5 \text{ V}, \quad Q_1 = -4 \mu\text{C}, \quad Q_2 = -1 \mu\text{C}, \quad Q_3 = 5 \mu\text{C}$$

1.136.

1.137.

$$C_{\text{nad.}} = \frac{1 + \sqrt{5}}{4} C, \text{ kjer je } C \text{ kapacitivnost posameznih kondenzatorjev v verigi.}$$

1.138. $C_{AB} \doteq 1,82 \mu\text{F}$

1.139. Na kondenzatorju, med čigar oglišči je priključena napetost 1 kV, je naboj 1 mF, na štirih kondenzatorjih so naboji približno 357,14 μF , na dveh približno 285,71 μF , na enem približno 142,86 μF in na štirih približno 71,43 μF .

1.140. Nadomestna kapacitivnost med ogliščema robu je $C_{\text{rob}} \doteq 5,14 \mu\text{F}$, med ogliščema ploskovne diagonale $C_{\text{plos.}} = 4 \mu\text{F}$ ter med ogliščema prostorske diagonale $C_{\text{pros.}} = 3,6 \mu\text{F}$.

1.141.

1.142. $W_{e, \text{diel.}}/W_{e, \text{zrak}} = 1/2$

1.143.

1.144.

1.145.

1.146. Referenčne smeri napetosti na kondenzatorjih izberimo (npr.) navzdol.

$$U_1 = 50 \text{ V}, \quad U_2 = 100 \text{ V}, \quad U_3 = U_4 = 75 \text{ V}$$

1.147. $W_e = 2 \mu\text{J}$

$$Q_{30 \text{ V}} = 720 \text{ nC}, \quad Q_{8 \text{ V}} = 420 \text{ nC}$$

1.148. $C_0/4$ in $3C_0/4$

1.149. $Q = 2 \text{ nC}$

1.150. $U_g = 11 \text{ V}$, $C = 5 \mu\text{F}$

1.151. $C/C_1 = k/(1 - 2k)$

1.152. $C_x = 4C$

1.153. $W_e \doteq 32,3 \text{ kJ}$

1.154. $\Delta W_e/W_e = 140 \%$

1.155. $\Delta Q = 3Q/5$

$$-\Delta W_e/W_e = 60 \%$$

1.156.

$$\frac{W_{e, \text{konč.}}}{W_{e, \text{zač.}}} = \frac{C_1(C_1 - C_2)^2}{(C_1 + C_2)^3}$$

1.157. $C_2 = C_3 = 4C_1$

1.158.

2 Časovno konstantno tokovno polje

2.1. Specifično prevodnost toroida označimo z γ .

a) Pravokotni presek; notranji polmer toroida označimo z a , zunanji z b ter širino preseka s c :

$$R = \frac{2\pi}{\gamma c \ln(b/a)}$$

b) Trapezni presek; notranji polmer toroida označimo z a , zunanji z b , širino manjše osnovice preseka s c ter večje z d :

$$R = \frac{2\pi}{\gamma} \left(d - c + \frac{bc - ad}{b - a} \ln \frac{b}{a} \right)^{-1}$$

2.2.

$$U = \frac{I}{2\pi\gamma l} \ln \frac{b}{a}$$

2.3. $\Delta R/R \doteq 21,7\%$

2.4. $\Delta R/R \doteq 22,5\%$

2.5.

$$R = \frac{1}{\pi\gamma h} \ln \frac{d + \sqrt{d^2 - d_1^2}}{d_1}$$

2.6.

$$R = \frac{1}{\pi\gamma d} \left(\ln \frac{a}{d_1} + 1 \right)$$

2.7. $R \doteq 110 \mu\Omega$

2.8.

$$V(\rho) \doteq 796 \text{ V} \cdot \ln \frac{2 \text{ m} + \sqrt{(2 \text{ m})^2 + \rho^2}}{\rho}$$

$$U_{\text{koraka}}(\rho) = V(\rho + 0,75 \text{ m}) - V(\rho) \doteq 796 \text{ V} \cdot \ln \frac{2 \text{ m} + \sqrt{(2 \text{ m})^2 + (\rho + 0,75 \text{ m})^2}}{2 \text{ m} + \sqrt{(2 \text{ m})^2 + \rho^2}}$$

2.9.

2.10. $R \doteq 92,8 \text{ m}\Omega$

2.11.

2.12. $W \doteq 28,3 \text{ kJ}$

2.13.

2.14.

$$J(x) = \frac{AI}{b \ln(Aa/B + 1)} \frac{1}{Ax + B}$$

2.15.

$$R = \frac{c\rho_0\alpha(\vartheta_2 - \vartheta_1)}{ab} \left(\ln \frac{1 + \alpha(\vartheta_2 - \vartheta_0)}{1 + \alpha(\vartheta_1 - \vartheta_0)} \right)^{-1}$$

$$\gamma(x) = f(x) \implies R = \frac{c}{b} \left(\int_0^a f(x) dx \right)^{-1}$$

$$\gamma(x) = \sin(\pi x/a) \implies R = \frac{\pi c}{2ab}$$

2.16. $x(V = 10 \text{ V}) \doteq 6,20 \text{ mm}$ 2.17. Površino preseka spirale označimo s S , njeno specifično prevodnost pa z γ :

$$R = \frac{1}{\gamma S} \int_0^{2\pi N} f(\varphi) d\varphi$$

a) Arhimedova spirala: $\rho(\varphi) = a\varphi$

$$R = \frac{2\pi^2 N^2 a}{\gamma S}$$

b) Logaritmična spirala: $\rho(\varphi) = ae^{k\varphi}$

$$R = \frac{a(e^{2\pi k N} - 1)}{\gamma k S}$$

2.18.

$$R = \frac{1}{\pi\gamma h} \ln \frac{d + \sqrt{d^2 - 4a^2}}{2a}$$

2.19. $J_{\text{izg.}}(\rho \rightarrow 1 \text{ cm} - 0) \doteq 83,1 \mu\text{A}/\text{m}^2$, $I_{\text{izg.}} \doteq 52,2 \text{ mA}$

$$P_{\text{izg.}} \doteq 522 \text{ W}$$

3 Enosmerna električna vezja

3.1. $n > 20$

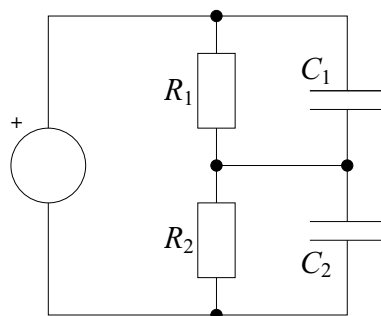
3.2. $k_1 \doteq 2,62$

3.3. $R_n = 8,55 \Omega$

3.4. $R_{AB} = 0,55 R$, $R_{AC} = 0,75 R$, $R_{AD} = 0,8 R$, $R_{AS} = 0,45 R$

3.5. $R_{\text{nad.}}(\varphi) = 4 \frac{\varphi}{\pi} \left(1 - \frac{\varphi}{\pi}\right) R$

3.6. $R_1 = \frac{l_1}{\gamma_1 A}$, $R_2 = \frac{l_2}{\gamma_2 A}$, $C_1 = \frac{\varepsilon_1 A}{l_1}$, $C_2 = \frac{\varepsilon_2 A}{l_2}$



Uporovni delilnik.

3.7. $R_1 = R_S \cdot (a - 1)(2 - a)$
 $R_2 = R_S \cdot \frac{2 - a}{a - 1}$,

kjer $a = U/U_S$ in $R_S = U_S^2/P_S$.

Rešitev je smiselna v območju $a \in (1, 2)$, torej za $U_S < U < 2U_S$.

3.8. $\frac{U_b}{U} = \frac{a}{1 - l + a/l}$
 $\frac{P_b}{P_{\text{cel.}}} = \frac{l}{l + a}$,

kjer $a = 1/k$.

3.9. $R = 10 \Omega$

3.10. $P_1 = \frac{30}{8} \text{ W}$

$$P_5 = -\frac{25}{8} \text{ W}$$

$$P_{10} = \frac{70}{8} \text{ W}$$

3.11. $P_1 = 36 \text{ W}$

3.12. $P_{4a} = 16 \text{ W}$, $P_{4b} = 4 \text{ W}$, $P_{10a} = 0 \text{ W}$, $P_{10b} = 40 \text{ W}$, $P_2 = 8 \text{ W}$

3.13. $P_z = 15 \text{ W}$, $P_5 = 5 \text{ W}$, $P_{20} = 5 \text{ W}$,
 $P_{1A} = 10 \text{ W}$, $P_{15V} = 15 \text{ W}$

- 3.14. $R_1 = 1 \text{ k}\Omega$
- 3.15. $R = 3 \Omega, P_{\max} = 27 \text{ W}$
- 3.16.
$$\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} a & -b \\ -b & a \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{a^2 - b^2} \begin{bmatrix} a & b \\ b & a \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix},$$
kjer sta $a = \frac{1}{2}(R_1 + R_2) = 2 \Omega$ in $b = \frac{1}{2}(R_1 - R_2) = -1 \Omega$.
- 3.17. $I_2 = \frac{2}{7} \text{ mA}$
- 3.18. $I = 1,2 \text{ mA}$
- 3.19.
- 3.20. $U_{***} = 100 \text{ V}$
- 3.21.
- 3.22. $I_1 = -0,4 \text{ A}, I_2 = 3,6 \text{ A}$
- 3.23. $U_1 = -600 \text{ V}, U_2 = 2400 \text{ V}$
- 3.24. $V_A = 80 \text{ V}, J_\alpha = 1 \text{ A}, P_g = 1170 \text{ W}$
- 3.25. $P_{\max} = 80 \text{ W}$
- 3.26. $R_1 = 1 \Omega, R_2 = 9 \Omega$
- 3.27. $P_{\max} = 6,25 \text{ W}$
- 3.28. $U_{\text{Th}} = 60 \text{ V}, I_N = 3 \text{ A}, R_N = R_{\text{Th}} = 20 \Omega$
- 3.29. $U_{\text{Th}} = -15 \text{ V}, I_N = -1 \text{ A}, R_{\text{Th}} = R_N = 15 \Omega$
- 3.30. $U_{\text{Th}} = -1 \text{ V}, R_{\text{Th}} = 4 \Omega$ (glej nalogo 3.27)
- 3.31. Vsaki od žarnic vežemo vzporedno en uporovni element. Od leve proti desni imajo vrednosti $R_1 = 4,8 \Omega, R_2 = 12 \Omega, R_3 = \infty$ (ne vežemo elementa, odprti sponki).
- 3.32. $R_x = 55 \Omega$
- 3.33. $P_3 = -0,3 \text{ W}, P_{15} = 27 \text{ W}$
- 3.34. $P_{10} = 40 \text{ W}, P_{20a} = 180 \text{ W}, P_{20b} = 20 \text{ W}$
- 3.35. $x \doteq 3,39 \text{ km}$
- 3.36. zaporedna vezava: $R_g = 25 \Omega, U_{g1} = 30 \text{ V}, U_{g2} = 10 \text{ V}$
vzporedna vezava: $R_g = 4 \Omega, U_{g1} = 12 \text{ V}, U_{g2} = -4 \text{ V}$
Dve vrednosti napetosti dobimo zaradi dveh možnosti vezave virov, stik in protistik.
- 3.37. $P_{10} = 4 \text{ W}$
- 3.38. $I_0 = 0,9 \text{ A}$
- 3.39. $P_{\max} = 98 \text{ W}$
- 3.40. $R_1 = R_2 = \frac{3}{2}(7 \pm 3\sqrt{5}) \Omega, R_3 = \frac{3}{2}(3 \pm \sqrt{5}) \Omega$
- 3.41. $Q = 7,2 \text{ MJ}$

3.42. $R_x = 7,5 \Omega, I_{A1} = \frac{1}{3} \text{ A}, I_{A2} = -3 \text{ A}$

3.43. $P_{10} = 50 \text{ W}, P_5 = 40 \text{ W}$

3.44. $R_1 = \frac{2}{3}R_b, R_2 = R_b$

$$R_1 = \frac{a}{1-ab}R_b, R_2 = \left[\frac{1}{b} - 1\right]R_b$$

3.45. $R_N = \frac{1}{2}(R_1 + R_2), I_N = \frac{R_2 - R_1}{R_2 + R_1}I_g$

3.46. $\begin{bmatrix} U_1 \\ U_2 \end{bmatrix} = \begin{bmatrix} 9 & 6 \\ 6 & 6 \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}, \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{2} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ U_2 \end{bmatrix}, \begin{bmatrix} U_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1 & -3 \\ \frac{1}{6} & -\frac{3}{2} \end{bmatrix} \cdot \begin{bmatrix} U_1 \\ I_1 \end{bmatrix}$

3.47.

3.48. $R_x = 20 \Omega$

3.49.

3.50.

3.51. Po metodi spojiščnih potencialov velja v poljubni točki kabla (za infinitezimalno majhne prispevke tokov) diferencialna enačba $V''(x) = \lambda^2 V(x)$, kjer je $\lambda = \sqrt{rg}$, x pa koordinata vzdolž kabla. Rešitev enačbe poiščemo z nastavkom $V(x) = Ae^{\lambda x} + Be^{-\lambda x}$. Neznani konstanti A in B določimo iz robnih pogojev, ki sta pri prvih dveh delih naloge $V(x=0) = U_g$ in $V(x \rightarrow \infty) = 0$, pri zadnjem pa $V(x=0) = U_g$ in $V(x=l) = 0$, kjer je $l = x_k$ (razdalje x_k so rezultat drugega dela naloge).

a) $V(x) = U_g \cdot e^{-\lambda x}$

$$I_0 = -\frac{1}{r} \frac{dV}{dx} \Big|_{x=0} = \sqrt{\frac{g}{r}} U_g \implies R_0 = \sqrt{\frac{r}{g}}$$

b) $\frac{V(x)}{U_g} = k \implies x_k = -\frac{1}{\lambda} \ln k, k \in [0,9, 0,8, \dots, 0,1]$

k	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,1
U/U_g	90 %	80 %	70 %	60 %	50 %	40 %	30 %	20 %	10 %
λx_k	0,1054	0,2231	0,3567	0,5108	0,6931	0,9163	1,204	1,6094	2,3026

c) $V(x) = U_g \cdot \frac{\text{sh } \lambda(l-x)}{\text{sh } \lambda l}$

$$R_{k0}(l = x_k) = \sqrt{\frac{r}{g}} \text{th} \left(\ln \frac{1}{k} \right)$$

k	0,9	0,8	0,7	0,6	0,5	0,4	0,3	0,2	0,1
U/U_g	90 %	80 %	70 %	60 %	50 %	40 %	30 %	20 %	10 %
λl	0,1054	0,2231	0,3567	0,5108	0,6931	0,9163	1,204	1,6094	2,3026
$\frac{R_{k0}}{\sqrt{r/g}}$	0,105	0,2195	0,3423	0,4706	0,6	0,7241	0,8349	0,9231	0,9802

4 Magnetostatično polje

4.1.

$$\mathbf{E} = \mathbf{e}_\rho E_\rho(\rho) = \mathbf{e}_\rho \frac{\rho_e d^2}{8\varepsilon_0 \rho}, \quad \mathbf{B} = \mathbf{e}_\varphi B_\varphi(\rho) = \mathbf{e}_\varphi \frac{\mu_0 \rho_e d^2 v}{8\rho} \implies \mathbf{B} = \frac{1}{c_0^2} \mathbf{v} \times \mathbf{E}$$

4.2. Izberimo koordinatni sistem (npr.) tako, da izhodišče sovpada s središčem tokovne zanke, da je os X vzporedna s stranico dolžine a (os Y pa s stranico dolžine b) ter da se smer osi Z po desnem pravilu ujema s smerjo toka.

$$\mathbf{B} = \mathbf{e}_z \frac{\mu_0 I}{4\pi} \left[\frac{1}{b/2 - y} \left(\frac{a/2 - x}{\sqrt{(a/2 - x)^2 + (b/2 - y)^2}} + \frac{a/2 + x}{\sqrt{(a/2 + x)^2 + (b/2 - y)^2}} \right) + \frac{1}{b/2 + y} \left(\frac{a/2 - x}{\sqrt{(a/2 - x)^2 + (b/2 + y)^2}} + \frac{a/2 + x}{\sqrt{(a/2 + x)^2 + (b/2 + y)^2}} \right) + \frac{1}{a/2 - x} \left(\frac{b/2 - y}{\sqrt{(a/2 - x)^2 + (b/2 - y)^2}} + \frac{b/2 + y}{\sqrt{(a/2 - x)^2 + (b/2 + y)^2}} \right) + \frac{1}{a/2 + x} \left(\frac{b/2 - y}{\sqrt{(a/2 + x)^2 + (b/2 - y)^2}} + \frac{b/2 + y}{\sqrt{(a/2 + x)^2 + (b/2 + y)^2}} \right) \right]$$

4.3. Koordinatni sistem izberimo (npr.) tako, da je os Z pravokotna na ravnino tokovne zanke, da se njena smer po desnem pravilu ujema s smerjo toka ter da je izhodišče v težišču zanke.

$$\mathbf{B}(T) = \mathbf{e}_z \frac{9\mu_0 I}{2\pi a}$$

$$\mathbf{B}(T) = \mathbf{e}_z B_z(z) = \mathbf{e}_z \frac{3\mu_0 I a^2}{8\pi \sqrt{3z^2 + a^2} (z^2 + a^2/12)}$$

4.4. Vektor \mathbf{B} je pravokoten na ravnino elipse, njegova smer je po desnem pravilu povezana s smerjo toka v ovoju in njegova velikost je $B = \frac{\mu_0 I a}{2b^2}$.

4.5. Koordinatni sistem izberimo (npr.) tako, da os X kaže v desno, os Y pa gor.

$$\mathbf{f}(T) = -\mathbf{e}_y \frac{\mu_0 I^2}{4\pi a}$$

4.6. Koordinatni sistem izberimo (npr.) tako, da os X kaže v desno, os Y pa gor. Če je točka T na spodnjem delu tokovodnika, potem je sila

$$\mathbf{f}_m(T) = -\mathbf{e}_y \frac{\mu_0 I^2}{4\pi a} \frac{1 - \cos \alpha}{\sin \alpha}.$$

4.7. a) Enakostranični trikotnik: poprečna sila na dolžinski meter posamezne vrvi je enaka $\langle f_m \rangle = \sqrt{3}/10 \text{ N/m} \doteq 0,173 \text{ N/m}$; njena smer je stran od težišča enakostraničnega trikotnika (poprečna sila je odbojna).

b) Vodoravno: poprečna sila na dolžinski meter srednje vrvi je enaka $\mathbf{0}$, zunanjih pa $\langle f_m \rangle = 0,15 \text{ N/m}$; po smeri je sila spet odbojna.

4.8.

$$f_m = \frac{\mu_0 I^2}{8a}$$

4.9. Sila je odbojna, njena velikost na dolžinski meter je

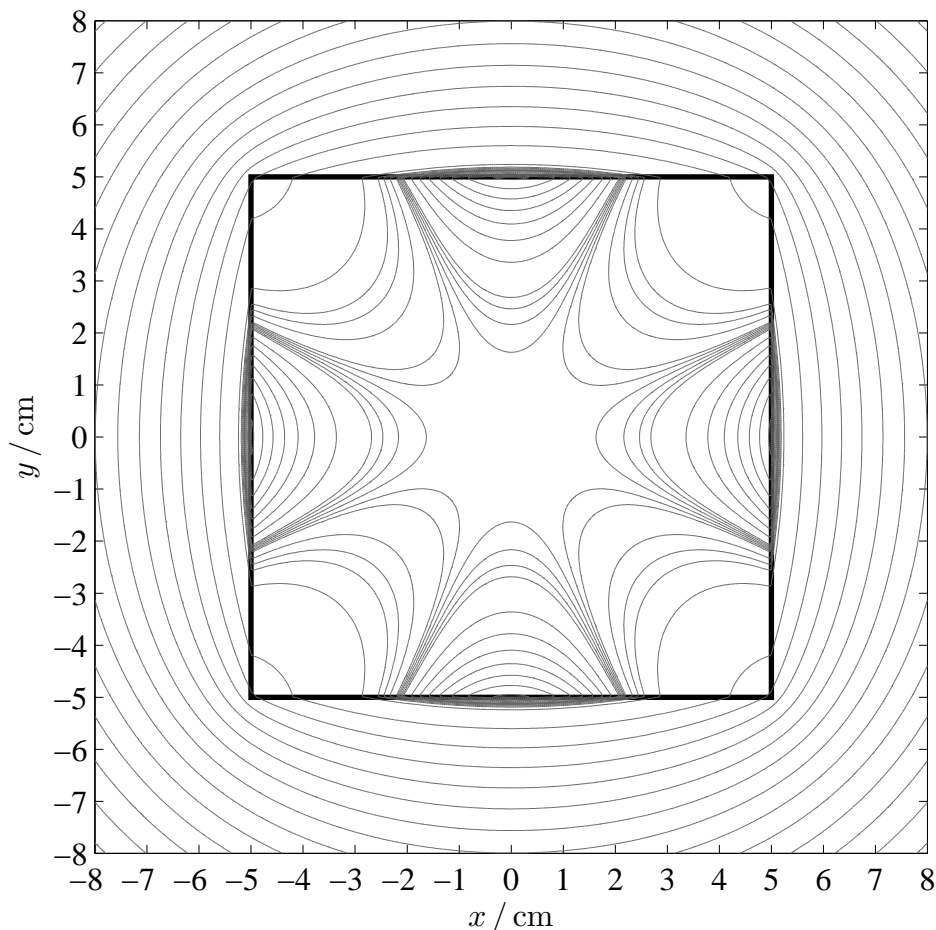
$$f_m = \frac{\mu_0 I^2}{2\pi a} \ln \frac{27}{16}.$$

4.10. $\mathbf{B}(T_1) = \mathbf{e}_y \frac{\mu_0 I}{8\pi a} \left[2 \left(\arctan \frac{3}{2} - \arctan \frac{1}{2} \right) - \ln \frac{13}{5} \right]$

Če sta $I = 100$ A in $a = 10$ cm, potem velja:

$$\mathbf{B}(T_1) \doteq \mathbf{e}_y 4,14 \mu\text{T}, \quad \mathbf{B}(T_2) \doteq (\mathbf{e}_x 8,57 - \mathbf{e}_y 8,57) \mu\text{T}, \quad \mathbf{B}(T_3) \doteq -\mathbf{e}_y 237 \mu\text{T} \quad \text{in} \quad \mathbf{B}(T_4) \doteq (\mathbf{e}_x 139 - \mathbf{e}_y 139) \mu\text{T}.$$

Prerezna slika pretočnic magnetnega pretoka za primer, ko je $a = 10$ cm:



4.11.

$$\mathbf{B}(T) = \mathbf{e}_y \frac{\mu_0 I}{4\pi a}, \quad \mathbf{B}(T_-) = \mathbf{e}_y \frac{3\mu_0 I}{4\pi a}, \quad \mathbf{B}(T_+) = -\mathbf{e}_y \frac{\mu_0 I}{4\pi a}$$

$$\mathbf{B}(T_1) = -\mathbf{e}_y \frac{\mu_0 I}{\pi^2 a}, \quad \mathbf{B}(T_2) = -\mathbf{e}_y \frac{\mu_0 I}{4\pi a}$$

4.12.

$$\mathbf{B}(T_0) = -\mathbf{e}_x \frac{2\mu_0 I}{\pi^2 a}$$

4.13.

- a) Trapezna ploskev; oddaljenosti osnovic trapeza do osi tokovodnika označimo z ρ_1 in ρ_2 , kot med podaljškoma bočnih stranic pa z α .

$$\phi = \frac{\mu_0 I}{2\pi} (\rho_2 - \rho_1) \tan \alpha$$

- b) Krožna ploskev; polmer krožne ploskve označimo z ρ_0 , pravokotno oddaljenost nje-nega središča do osi tokovodnika pa z ρ_1 .

$$\phi = \mu_0 I \left(\rho_1 - \sqrt{\rho_1^2 - \rho_0^2} \right)$$

4.14.

- a) Primer zgoraj levo:

$$B(T) = \frac{\mu_0 I}{2a} (1 + 1/\pi), \text{ smer polja je iz lista.}$$

- b) Primer zgoraj desno:

$$B(T) = \frac{\mu_0 I}{4a} (1 + 2/\pi), \text{ smer polja je v list.}$$

- c) Primer spodaj:

$$B(T) = \frac{\mu_0 I}{4a}, \text{ smer polja je iz lista.}$$

4.15.

$$\mathbf{B}(z) = \mathbf{e}_z \frac{\mu_0 N I}{2(b-a)} \left[\ln \left| \tan \frac{\arctan(b/z) + \pi/2}{2} \right| - \ln \left| \tan \frac{\arctan(a/z) + \pi/2}{2} \right| + \sin [\arctan(a/z)] - \sin [\arctan(b/z)] \right]$$

$$\int_{z_1=0}^{z_2 \rightarrow \infty} \mathbf{B}(z) \cdot d\mathbf{l} = \mu_0 N I / 2$$

4.16.

Koordinatni sistem izberimo (npr.) tako, da os Z sovpada z osjo tuljave, da je njena smer po desnem pravilu povezana s smerjo toka ter da je koordinatno izhodišče v središču tuljave.

$$\mathbf{B}(z) = \mathbf{e}_z \frac{\mu_0 N I}{2l(b-a)} \left[(z + l/2) \ln \frac{b + \sqrt{b^2 + (z + l/2)^2}}{a + \sqrt{a^2 + (z + l/2)^2}} + (-z + l/2) \ln \frac{b + \sqrt{b^2 + (-z + l/2)^2}}{a + \sqrt{a^2 + (-z + l/2)^2}} \right]$$

4.17. $I \doteq 32,0 \text{ kA}$

4.18.

$$f = \frac{\mu_0 I^2}{56\pi a}$$

- 4.19.** Polmera vodnikov označimo z a , njuno medosno razdaljo pa z d . Koordinatni sistem izberimo (npr.) tako, da je os Z na sredi med osema vodnikov in da os X kaže v desno ter os Y gor.

$$T_0(x_0 = -\sqrt{a^2 + (d/2)^2}, y_0 = 0), \quad T'_0(x'_0 = \sqrt{a^2 + (d/2)^2}, y'_0 = 0)$$

$$\phi_{T_0, T'_0}/l = \frac{\mu_0 I}{\pi} \left[\frac{-d^2 + 2d\sqrt{a^2 + (d/2)^2}}{4a^2} + \ln \frac{d/2 + \sqrt{a^2 + (d/2)^2}}{a} \right]$$

$$\phi_{T_1, T'_1}/l = \frac{\mu_0 I}{\pi} \ln \frac{d-a}{a}$$

$$\phi_{T_2, T'_2}/l = \frac{\mu_0 I}{\pi} \ln \frac{\sqrt{a^2 + d^2}}{a}$$

$$\phi_{T_3, T'_3}/l = \frac{\mu_0 I}{\pi} \ln \frac{d+a}{a}$$

S stališča določanja induktivnosti je (pravi) fluks dvovoda poprečje vseh možnih.

- 4.20.** Koordinatni sistem izberimo (npr.) tako, da os Z sovpada z osjo zunanje stene vodnika ter da os X kaže v desno, Y pa gor.

a) V luknji:

$$\mathbf{B} = \mathbf{e}_y \frac{\mu_0 J}{2} e.$$

b) V prerezu:

$$\mathbf{B} = \mathbf{e}_x \frac{\mu_0 J}{2} y \left(\frac{a^2}{(x-e)^2 + y^2} - 1 \right) + \mathbf{e}_y \frac{\mu_0 J}{2} \left(x - \frac{(x-e)a^2}{(x-e)^2 + y^2} \right).$$

c) Izven tokovodnika:

$$\mathbf{B} = \mathbf{e}_x \frac{\mu_0 J}{2} y \left(\frac{a^2}{(x-e)^2 + y^2} - \frac{b^2}{x^2 + y^2} \right) + \mathbf{e}_y \frac{\mu_0 J}{2} \left(\frac{xb^2}{x^2 + y^2} - \frac{(x-e)a^2}{(x-e)^2 + y^2} \right).$$

$$\phi_{T_1, T_2} = \frac{\mu_0 J l a^2}{2} \ln \frac{b+e}{b-e}$$

- 4.21.** $\phi_{T_1, T_2} = 0,4 \mu\text{Wb}$

4.22.

$$\phi = \frac{\mu_0 I l}{2\pi} \left(\frac{1}{2} - \frac{1}{e^2 - 1} \right)$$

4.23.

$$\phi/l = \frac{\mu_0 I}{2\pi} \left(\frac{1}{2} - \frac{\rho_n^2}{\rho_z^2 - \rho_n^2} \ln \frac{\rho_z}{\rho_n} \right)$$

4.24.

$$\oint_{\mathcal{L}_1} \mathbf{B} \cdot d\mathbf{l} = 0$$

$$\oint_{\mathcal{L}_2} \mathbf{B} \cdot d\mathbf{l} = -2\mu_0 I$$

$$\oint_{\mathcal{L}_3} \mathbf{B} \cdot d\mathbf{l} = -\mu_0 I$$

$$\oint_{\mathcal{L}_4} \mathbf{B} \cdot d\mathbf{l} = 3\mu_0 I$$

4.25. Izberimo (npr.) valjni koordinatni sistem z osjo Z v osi tokovodnika. Zaradi nehomogene porazdelitve temperature je tudi tokovna gostota nehomogena (radialna funkcija), $\mathbf{J} = e_z J_z(\rho)$. Polmer tokovodnika označimo z ρ_0 , njegov tok pa z $I (= 2\pi \int_0^{\rho_0} J_z(\rho') \rho' d\rho')$.

$$\mathbf{B}(\rho) = \begin{cases} e_\varphi (\mu_0/\rho) \int_0^\rho J_z(\rho') \rho' d\rho' & , \rho \leq \rho_0 \\ e_\varphi \mu_0 I / (2\pi\rho) & , \rho > \rho_0 \end{cases}$$

4.26.

$$\mathbf{B}(T_0) = e_\varphi \frac{\mu_0 I_0}{8\pi\rho_0}$$

4.27.

$$\phi/l = \frac{\mu_0 I}{2\pi} \left[\ln(b/a) + (1 + (b/a)^2) \ln \sqrt{1 + (a/b)^2} \right]$$

4.28.

$$B(T) = \frac{\mu_0 I}{2\pi a} (\sqrt{2} - 1)$$

4.29.

$$\mathbf{B}(T_1) = e_\varphi \frac{\mu_0 I}{\pi h}$$

$$\mathbf{B}(T_2) = e_\varphi \frac{\mu_0 I}{\pi h} \left(1 + \frac{\sqrt{2}}{4} - \frac{3}{2\sqrt{10}} \right) \doteq e_\varphi \frac{\mu_0 I}{\pi h} 0,879$$

$$\mathbf{B}(T_3) = e_\varphi \frac{\mu_0 I}{\pi h} \left(1 - \frac{2}{\sqrt{17}} \right) \doteq e_\varphi \frac{\mu_0 I}{\pi h} 0,515$$

$$\mathbf{B}(T_4) = e_\varphi \frac{\mu_0 I}{\pi h} \left(1 - \frac{\sqrt{2}}{4} - \frac{5}{2\sqrt{26}} \right) \doteq e_\varphi \frac{\mu_0 I}{\pi h} 0,156$$

4.30.

4.31.

$$A_m = \frac{\mu_0 I I_1 a}{2\pi} \ln \frac{3}{2}$$

4.32. $\phi_{\text{Fe}}/l = 800 \mu\text{Wb}/\text{m}$, $\phi_{\text{Al}}/l \doteq 29,1 \mu\text{Wb}/\text{m}$ 4.33. $\mathbf{B}(y > 0) \doteq (188, 754, 188) \mu\text{T}$ 4.34. $\mathbf{H}(x > 0) = (10, 1, 1) \text{A}/\text{m}$

$$\mathbf{B}_{\text{tang.}}(x > 0) = \mathbf{0} \implies \mathbf{K}(x = 0) = (0, 1, -1) \text{A}/\text{m}$$

4.35. $B_z(z > 0) \doteq 0,314 \text{mT}$, $\mathbf{K}(z = 0) = (50, 100, 0) \text{A}/\text{m}$

4.36. Magnetno polje v okolici tračnega vodnika je določeno v zgledu 41.4 v učbeniku Anton R. Sinigoj "Osnove elektromagnetike".

4.37.

$$F > \frac{24\mu_0 I_1 I_2 \pi a_1^2}{25\sqrt{5}a_2^2}$$

4.38. Navor na prvo zankico je

$$\mathbf{M}_1 = \frac{\mu_0}{4\pi R_{12}^3} \mathbf{m}_1 \times \left(\frac{3(\mathbf{m}_2 \cdot \mathbf{R}_{12})\mathbf{R}_{12}}{R_{12}^2} - \mathbf{m}_2 \right),$$

kjer je \mathbf{R}_{12} distančni vektor med točkama T_1 in T_2 . Podobno je navor na drugo zankico

$$\mathbf{M}_2 = \frac{\mu_0}{4\pi R_{12}^3} \mathbf{m}_2 \times \left(\frac{3(\mathbf{m}_1 \cdot \mathbf{R}_{12})\mathbf{R}_{12}}{R_{12}^2} - \mathbf{m}_1 \right).$$

4.39. $B \doteq 1,23 \text{ T}$, $\phi \doteq 0,701 \text{ mWb}$, $M \doteq 978 \text{ kA/m}$ 4.40. $\phi \doteq 56 \text{ }\mu\text{Wb}$

4.41.

4.42. $H \doteq -7,96 \text{ kA/m}$, $M \doteq 406 \text{ kA/m}$ 4.43. $I \doteq 0,98 \text{ A}$

Če bi bili stebri različnih presekov, bi morali poznati površine teh presekov, da bi lahko rešili nalogo.

4.44. $\phi_1 : \phi_2 : \phi_3 = (-16) : 27 : (-11)$ 4.45. $\theta \doteq 6,67 \text{ kA}$ 4.46. Referenčno smer gostote (B_1) v levem stebru izberimo (npr.) navzgor, v srednjem (B_3) in desnem (B_2) stebru pa navzdol.

$$B_1 \doteq 0,251 \text{ T}, \quad B_2 \doteq 0,126 \text{ T}, \quad B_3 \doteq 0,126 \text{ T}$$

4.47. $I = 0,1 \text{ A}$ 4.48. $I \doteq 0,5 \text{ A}$ 4.49. $H_1 = H_2$

$$B_1 = -B_2 \left((b/a)^2 - 1 \right) = -\mu_0 H_1 \left((b/a)^2 - 1 \right), \quad B_2 = \mu_0 H_2$$

$$M_1 = -H_1 (b/a)^2$$

4.50.

$$I_1 = \frac{\mu_r - 1}{\mu_r + 1} I, \quad I_2 = \frac{2\mu_r}{\mu_r + 1} I$$

Sila na tokovodnik je usmerjena pravokotno proti feromagnetni steni (privlačna sila), njena velikost na dolžinski meter vodnika je

$$f_m = \mu_0 \frac{\mu_r - 1}{\mu_r + 1} \frac{I^2}{4\pi a}.$$

4.51.

4.52. Sila na tokovodnik je usmerjena proti feromagnetnem vogalu (privlačna sila), njena velikost na dolžinski meter vodnika je

$$f_m = \frac{3\sqrt{2}\mu_0 I^2}{8\pi a}.$$

5 Dinamično elektromagnetno polje

$$5.1. \quad P = \frac{2}{R}(lB\omega d)^2$$

$$5.2. \quad \Delta Q = \frac{\mu_0 \pi I_2 a_1^2}{2R_1 a_2}$$

$$5.3. \quad Q = (\omega BS \sin \alpha)^2 \frac{(\Delta t)}{2R} \doteq 4,5 \text{ J}$$

$$I_C = \omega^2 SBC \cos(\omega t + \varphi_0) \sin \alpha$$

$$5.4. \quad U = \frac{1}{40} \pi n_1 B a^2$$

$$5.5. \quad i(t) = \frac{1}{R} \left[U_0 - a B_0 v_0 \left(1 - \frac{1}{e} \right) \exp\left(-\frac{v_0 t}{b}\right) \right]$$

5.6.

$$5.7. \quad u_{\text{ind}}(t) = (\omega_1 - \omega_2) B_0 S \cdot \sin \left[(\omega_1 - \omega_2)t + \varphi_1 - \varphi_2 \right]$$

5.8.

$$5.9. \quad E_1 = \frac{1-b}{b+c} \cdot \frac{\omega \Phi_0}{2\pi a} \sin(\omega t)$$

$$E_2 = c \frac{b-1}{b+c} \cdot \frac{\omega \Phi_0}{2\pi a} \sin(\omega t)$$

$$E_{\text{ind}} = \frac{\omega \Phi_0}{2\pi a} \sin(\omega t)$$

$$5.10. \quad \bar{P} = \frac{1}{16} \pi \gamma h a^4 B_0^2 \omega^2$$

$$5.11. \quad \mathbf{J} = \mathbf{e}_\varphi J_\varphi(\varphi, t) = \frac{1}{2} \omega \gamma B_0 \cos(\omega t)$$

$$\frac{\bar{P}}{m} = \frac{1}{24 \rho_{\text{Fe}}} \gamma \omega^2 B_0 d^2$$

$$5.12. \quad \mathbf{F} = -\pi \gamma d a^2 B^2 \mathbf{v}$$

$$5.13. \quad \psi = -2\Phi_1 + 3\Phi_2 - \Phi_3 = -3\Phi_1 + 2\Phi_2 - 2\Phi_3$$

5.14.

$$5.15. \quad x_{1,2} = \pm d \sqrt{\frac{3}{2}}$$

$$5.16. \quad u_{\text{ind}} = 2\mu_0 \left(\rho_1 - \sqrt{\rho_1^2 - \rho_0^2} \right) \cdot I e^{-\alpha t} \left[\alpha \cos(\beta t + \gamma) + \beta \sin(\beta t + \gamma) \right]$$

$$5.17. \quad L_1 = \frac{\mu_0 l}{\pi} \left[\frac{1}{4} + \ln \frac{d\sqrt{d^2 + 4h^2}}{2h\rho_0} \right]$$

$$L_2 = \frac{\mu_0 l}{\pi} \left[\frac{1}{4} + \ln \frac{d(d+2h)}{2\rho_0\sqrt{h(h+d)}} \right]$$

$$\phi_1 = \frac{\mu_0 Il}{\pi} \ln \frac{\sqrt{d^2 + 4h^2} + d}{\sqrt{d^2 + 4h^2} - d}$$

$$\phi_2 = \frac{\mu_0 Il}{\pi} \ln \frac{d+h}{h}$$

5.18.

5.19. $u_2(t) \doteq -6,7 \cdot \sin(\omega t) \text{ mV}$

5.20. $u_{\text{ind}}(t) \doteq 16,8 \cdot \sin(\omega t + \alpha) \text{ V}$

5.21. $L = \frac{2N^2}{5R}$

5.22. $L_{12} = \frac{\mu_0 N_1 N_2 a_2}{2\pi} \ln \frac{r_0 + a_2/2}{r_0 - a_2/2}$

5.23. $L_{12} = 1 \text{ H}$

5.24. $M = \frac{1}{4\pi R_{12}^2} \left[3(\delta \mathbf{a}_1 \cdot \mathbf{R}_{12})(\delta \mathbf{a}_2 \cdot \mathbf{R}_{12}) - R_{12}^2 \delta \mathbf{a}_1 \cdot \delta \mathbf{a}_2 \right]$

5.25. $L_1 = L_2 = \frac{\mu_0 l}{\pi} \left[\ln \frac{d}{\rho_0} + \frac{1}{4} \right]$

$$L_{12} = \frac{\mu_0 l}{2\pi} \ln \frac{(2d + \Delta)\Delta}{(d + \Delta)^2},$$

$$R_1 = R_2 = \frac{2l + d}{\pi\gamma\rho_0^2}$$

5.26.

5.27. $k = 0,6$

5.28. $\frac{\bar{W}}{l} \doteq 29,9 \cdot 10^{-3} \frac{\text{J}}{\text{m}}$

5.29. $W \doteq 0,488 \text{ J}$

5.30. $W \doteq 9,91 \text{ J}$

5.31. $W \doteq 21,2 \text{ mJ}$

5.32. $W \doteq 25 \text{ mJ}$

5.33. $F \doteq 796 \text{ N}$

5.34. $\bar{F} \doteq 7,78 \text{ N}$

5.35. $IN \doteq 257 \text{ A}$

5.36. $\bar{F} \doteq 59,7 \text{ N}$

6 Električna vezja spremenljivih tokov

6.1. $\bar{I} = \frac{I_0}{\pi}(1 + \cos \alpha)$

$$I_{\text{ef}} = I_0 \sqrt{\frac{1}{2\pi} \left[\pi - \alpha + \frac{1}{2} \sin(2\alpha) \right]}$$

6.2. $u_L(t) = \omega L I_0 \cos(\omega t) = 10^4 \cos(\omega t) \text{ V}$

$$W_m(t) = 500 \sin(\omega t) \text{ J}$$

$$\bar{W}_m = 250 \text{ J}$$

6.3. $i_C(t) = \omega C U_0 \cos(\omega t) = 100 \cos(\omega t) \text{ A}$

$$Q_{\text{max}} = 0,1 \text{ C}$$

$$W_e(t) = 500 \sin^2(\omega t) \text{ J}$$

$$\bar{W}_e = 250 \text{ J}$$

6.4.

6.5. $u_C(t_1) = 3,68 \text{ V}$

6.6. $i(t) = \frac{Q}{RC} e^{-t/\tau}$, kjer $\tau = \frac{1}{2} RC$

za splošni primer $C_1 \neq C_2$ in $Q_1 \neq Q_2$:

$$i_a(t) = \pm \frac{2}{RC_1 C_2} (C_2 Q_1 + C_1 Q_2) e^{-t/\tau}$$

$$i_b(t) = \pm \frac{2}{RC_1 C_2} (C_2 Q_1 - C_1 Q_2) e^{-t/\tau}$$

6.7. $Q = \frac{R_B \lambda_1 \lambda_2}{2(\lambda_1 + \lambda_2)} \cdot \left[\frac{M U_g}{(R_2 + R_B) R_1} \right]^2$, kjer:

a) $M = k \sqrt{L_1 L_2}$

b) λ_1 in λ_2 sta korena kvadratne enačbe $a\lambda^2 + b\lambda + c = 0$ s koeficienti $a = L_1 L_2 - M^2$,
 $b = L_1(R_1 + R_B) + R_1 L_2$, $c = R_1(R_2 + R_B)M$

6.8. $i(t) = \frac{U_g/R}{\sqrt{1 + (\omega\tau)^2}} \left[\cos(\omega t + \varphi_g - \arctg(\omega\tau)) - \cos(\varphi_g - \arctg(\omega\tau)) e^{-t/\tau} \right]$, $\tau = \frac{L}{R}$

6.9. $t_0 \doteq 1064 \text{ s} \doteq 17,7 \text{ min}$

6.10. $i(t) = 2500 \cdot [2 - e^{-t/\tau}] \text{ A}$, kjer $\tau = 3 \text{ ms}$, $i(t_1 = 1 \text{ ms}) \doteq 3,2 \text{ kA}$

6.11. $i(t) = \frac{U_g}{R_1 + R_2} \left[1 + \frac{R_2}{R_1} e^{-t/\tau} \right] = \left[0,4 + 0,1 e^{-t/\tau} \right] \text{ A}$,

kjer $R_1 = 20 \Omega$, $R_2 = 5 \Omega$, $\tau = \frac{R_1 R_2 C}{R_1 + R_2}$

6.12. $U_a = U_0 e^{-T_2/\tau} \frac{1 - e^{-T_1/\tau}}{1 - e^{-(T_1+T_2)/\tau}}$

$$U_b = U_0 \frac{1 - e^{-T_1/\tau}}{1 - e^{-(T_1+T_2)/\tau}}$$

6.13.

6.14.

$$6.15. \quad u_1(t) = \frac{R_1}{R_1 + R_2} (1 - e^{-t/\tau})$$

$$u_2(t) = \frac{U_g}{R_1 + R_2} (R_2 - R_1 e^{-t/\tau}), \quad \tau = (C_1 + C_2) \frac{R_1 R_2}{R_1 + R_2}$$

6.16. $\underline{S} = (20 + j10) \mu\text{VA}$

6.17.

6.18. $u_R(t_0) = \pm 8.9 \text{ V}$

6.19. $\underline{Z} \doteq (775 + j1036) \Omega$

$\underline{Y} = \frac{1}{\underline{Z}} \doteq (463 - j619) \mu\text{S}$

$$6.20. \quad \underline{Z}(\omega) = \frac{1}{\omega L/R + R/(\omega L)} \left[\omega L + j \left(R - \frac{L}{RC} - \frac{R}{\omega^2 LC} \right) \right]$$

$$\omega' = \frac{R}{\sqrt{L(R^2 C - L)}}$$

$$\underline{Z}(\omega') = \frac{L}{RC}$$

6.21. $R = 5 \Omega$

6.22. $C \doteq 9, 11 \mu\text{F}, u(t_0) \doteq \pm 156 \text{ V}$

6.23.

6.24. $\bar{P} = 19, 4 \text{ kW}$

6.25. $W_m(t_0) = 1, 5 \mu\text{J}$

6.26. $C_1 \doteq 1, 47 \mu\text{F}, C_2 \doteq 11, 2 \mu\text{F}$

6.27. $\underline{Z}_{\text{vh}} = j10 \Omega$

6.28.

6.29. $X_C = 6 \Omega$

6.30. $\bar{P} = 352 \text{ W}$

6.31. $\underline{U} = (16 + j41) \text{ V}$

6.32. $\underline{Z} = (20 - j\frac{5}{6}) \Omega$

6.33. $\bar{P} = 1650 \text{ W}$

6.34. $I_1 = \sqrt{3}/2 \text{ A}$

6.35. $\underline{U}_{\text{Th}} = 0\text{V}, \underline{Z}_{\text{Th}} \doteq (1, 92 + j5, 44) \Omega$

6.36. $\underline{Z}_b = (0, 8 + j0, 4) \Omega, P_{\text{max}} = 2 \text{ W}$

6.37. $P \doteq 614 \text{ W}$

6.38. $P = 4 \text{ W}$

6.39. $\underline{Z}_b = (2, 5 - j3) \Omega, P_{\text{max}} = 1, 8 \text{ W}$

6.40. $C \in [80, 175] \mu\text{F}$

6.41.

- 6.42. $P \doteq 6,97 \text{ kW}$, $\eta \doteq 65,3 \%$
- 6.43.
- 6.44.
- 6.45. $R \doteq 62,8 \Omega$, $C \doteq 61,9 \mu\text{F}$
- 6.46. $\underline{Z}_b = 5(1 + j) \Omega$, $P_{\max} = 250 \text{ W}$
- 6.47. $\underline{S} = (165 + j95) \text{ VA}$
- 6.48. $U_V = 400 \text{ V}$
- 6.49. $\omega_0^2 = \frac{1}{LC - \frac{1}{2}(RC)^2}$
- $$U_V(\omega_0) = \frac{U_{\text{ef}}}{\sqrt{(1 - R^2 \frac{C}{L})R^2 \frac{C}{L}}}$$
- 6.50. $\omega_{0I}^2 = \frac{1}{C(L_2 - M^2/L_1)}$
- $$\omega_{0U}^2 = \frac{1}{L_2 C}$$
- 6.51. $\omega_0^2 = \frac{1}{LC} - (\frac{R}{L})^2$
- 6.52. $\omega_{0I} = \frac{n}{\sqrt{LC}}$
- 6.53.
- 6.54. $R = \frac{\omega L}{\sqrt{\omega^2 LC - 1}}$
- 6.55. $\underline{Z}_{\text{vh}} = \begin{cases} j \left[\frac{\omega L}{2} \mp \sqrt{\frac{\omega^2 L^2}{4} - \frac{L}{C}} \right] & ; \omega^2 LC \geq 4 \\ \sqrt{\frac{L}{C} - \frac{\omega^2 L^2}{4}} + j \frac{\omega L}{2} & ; \omega^2 LC < 4 \end{cases}$
- 6.56. $R_{\text{vh}} = R \left(\frac{n}{n+1} \right)^2$
- 6.57. $R = \frac{2}{\sqrt{3}} \Omega$, $X_C = 2 \Omega$
- 6.58. $\underline{S}_A = 0$, $\underline{S}_B = 1,45 \left(\frac{1}{2} - j \frac{\sqrt{3}}{2} \right) \text{ kVA}$, $\underline{S}_C = \underline{S}_B^*$
- 6.59. $I_A \doteq 7,34 \text{ A}$
- 6.60. $U_V \doteq 519 \text{ V}$
- 6.61. $k \doteq 0,268 \doteq \frac{1}{3,73}$
- 6.62. $\underline{Z} = 10 \angle 30^\circ \Omega$
- 6.63.
1. $I_{A1} = 5 \text{ A}$, $I_{A2} \doteq 8,66 \text{ A}$ (pozitivni sistem)
 2. $I_{A1} = 10 \text{ A}$, $I_{A2} = 0 \text{ A}$ (negativni sistem)
- 6.64. $U_{\text{mf}} \doteq 623 \text{ V}$
- 6.65. $I_A \doteq 94,6 \text{ A}$
- 6.66. $I_A \doteq 26,5 \text{ A}$, $P \doteq 5,7 \text{ kW}$
- 6.67. $C \doteq 2,33 \mu\text{F}$

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6.68. $I_A = \frac{2U_g}{R}$

6.69. $P_{b,\max} \doteq 3,82 \text{ MW}$

6.70. $R = 1 \text{ k}\Omega, L \doteq 5,5 \text{ H}, C \doteq 1,84 \text{ }\mu\text{F}$

6.71. $C \doteq \begin{cases} 1,03 \text{ }\mu\text{F}; & \text{zvezda} \\ 342 \text{ }\mu\text{F}; & \text{trikot} \end{cases}$